

# Challenges to the Generalized Chaplygin Gas Cosmology

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The generalized Chaplygin gas (GCG) model allows for an unified description of the cosmologically recent accelerated expansion of the Universe and of the evolution of energy density perturbations. This dark energy - dark matter unification is achieved through a rather exotic background fluid whose equation of state is given by  $p = -A/\rho^\alpha$ , where  $A$  is a positive constant. Observational constraints arising from bounds on the locations of the first few peaks and troughs of the Cosmic Microwave Background Radiation (CMBR) power spectrum from recent WMAP and BOOMERanG experiments are consistent with the model for  $\alpha \lesssim 0.6$  assuming that  $0 < \alpha \leq 1$ . Most recent Type-Ia Supernova data indicates however, that the range  $\alpha > 1$  must be considered.

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## I. INTRODUCTION

In this brief review I shall summarize some recent results obtained in the context of the GCG model and discuss some challenges this dark energy - dark matter unification model face. The results presented here largely rely on the work developed in Refs. [1, 2, 3, 4, 5, 6].

Cosmology is undergoing a particularly fruitful period. Recent precision measurements obtained by dedicated experiments on the Cosmic Microwave background Radiation (CMBR), supernova searches and large galaxy surveys allow for detailed comparisons with the theoretical models. It is remarkable that all available data can be accounted by the *Hot Big Bang Model* enriched with *Inflation*, a period of accelerated expansion in the very early Universe that reconciles cosmology with causality and that elegantly explains the origin of the observed Large Scale Structure of the Universe. However, in order to fully understand the observations, at least two additional new entities are required: *Dark Matter* and *Dark Energy*. Dark matter was originally proposed to explain the rotation curves of galaxies and it turns out to be a fundamental building block for structure formation at large scales. Dark energy corresponds to a smoothly distributed energy that cannot be related with any known form of matter and is required to explain the recently observed accelerated expansion of the Universe. Even though presumably the inflationary process has been caused by the the dynamics of a scalar field, the inflaton, and that the underlying structures behind dark energy and dark matter might also be scalar fields, these three concepts are apparently unrelated. However, in what concerns dark matter and dark energy, a scheme has emerged where an unification of these entities is possible through a perfect fluid description with an exotic equation of state:

$$p_{ch} = -\frac{A}{\rho_{ch}^\alpha}, \quad (1)$$

where  $A$  and  $\alpha$  are positive constants. Most of the work

on GCG cosmology has assumed that  $0 < \alpha \leq 1$ , however, more recently, in studying the latest Type-Ia Supernova data it has emerged that  $\alpha > 1$  values are preferred. This equation of state with  $\alpha = 1$  was first introduced in 1904 by the Russian physicist Chaplygin to describe aerodynamic processes [7]; its importance to cosmology was pointed out in Ref. [8]. The suggestion of its generalization for  $\alpha \neq 1$  was also proposed in Ref. [8] and the ensuing cosmology has been analyzed in Ref. [1]. It is remarkable that the Chaplygin equation of state has a well defined connection with string and brane theories (see Ref. [11] for a through review).

The idea that a cosmological model based on the Chaplygin gas could lead to the unification of dark energy and dark matter, was first advanced for the  $\alpha = 1$  case in Refs. [9, 10], and generalized to  $\alpha \neq 1$  in Ref. [1].

## II. THE MODEL

The reason why the cosmological features of the equation of state (1) are so interesting can be better appreciated after inserting it into the relativistic energy-momentum conservation equation, which yields for the evolution of the energy density [1]

$$\rho_{ch} = \left[ A + \frac{B}{a^{3(1+\alpha)}} \right]^{\frac{1}{1+\alpha}}, \quad (2)$$

where  $a$  is the scale-factor of the Universe and  $B$  an integration constant. It is easy to see that Eq. (2) is directly related with the observed accelerated expansion of the Universe as it automatically leads to an asymptotic phase where the equation of state is dominated by a cosmological constant,  $8\pi G A^{1/(1+\alpha)}$ , while at earlier times the energy density behaves as if dominated by non-relativistic matter. This dual behaviour is at the heart of the unification scheme provided by the GCG model. This unification can be achieved through an underlying description based on a complex scalar field model which admits

an inhomogeneous generalization. This generalization is shown to be consistent with standard structure formation scenarios [1, 9, 10]. It is clear that the GCG model corresponds to the  $\Lambda$ CDM model for  $\alpha = 0$  (and also  $A_s = 1$ ; see Eq. (9) below).

These remarkable properties make the GCG model an interesting alternative to models where the accelerated expansion of the Universe arises from an uncanceled cosmological constant or a rolling scalar field as in quintessence models.

From Eq. (1) one can see that, in principle, any positive  $\alpha$  values are admissible. The range  $0 < \alpha \leq 1$ , has been chosen so that the sound velocity ( $c_s^2 = \alpha A / \rho_{ch}^{1+\alpha}$ ) does not exceed the velocity of light, in the regime where the effective equation of state has the form of “soft” matter,  $p = \alpha \rho$ , in which case  $c_s^2 = \alpha$ . Notice however, that as described, in Ref. [6], one can accommodate the case  $\alpha > 1$  in a manifestly Lorentz invariant underlying theory that does not violate the dominant energy condition  $\rho + p \geq 0$ .

As already mentioned, at a more fundamental level, the GCG model can be described by a complex scalar field whose Lagrangian density can be written in the form of a generalized Born-Infeld Lagrangian density. This can be seen starting with the Lagrangian density for a massive complex scalar field,  $\Phi$ ,

$$\mathcal{L} = g^{\mu\nu} \Phi_{,\mu}^* \Phi_{,\nu} - V(|\Phi|^2) , \quad (3)$$

expressed in terms of its mass,  $M$ , as  $\Phi = (\phi/\sqrt{2}m) \exp(-iM\theta)$ . Considering the scale of the inhomogeneities as corresponding to the spacetime variations of  $\phi$  on scales greater than  $M^{-1}$ , then  $\phi_{,\mu} \ll M\phi$ , which, together with Eq.(1), yields to the relationship:

$$\phi^2(\rho_{ch}) = \rho_{ch}^\alpha (\rho_{ch}^{1+\alpha} - A)^{\frac{1-\alpha}{1+\alpha}} , \quad (4)$$

following that the Lagrangian density Eq. (3) assumes the form of a *generalized* Born-Infeld Lagrangian density:

$$\mathcal{L}_{GBI} = -A^{\frac{1}{1+\alpha}} \left[ 1 - (g^{\mu\nu} \theta_{,\mu} \theta_{,\nu})^{\frac{1+\alpha}{2\alpha}} \right]^{\frac{\alpha}{1+\alpha}} . \quad (5)$$

Notice that, for  $\alpha = 1$ , one recovers the exact Born-Infeld Lagrangian density.

Alternatively, the GCG model can be described by a minimally coupled scalar field,  $\varphi$ , with canonical kinetic energy term and a potential of the form [6]:

$$V = V_0 e^{3(\alpha-1)} \left[ \cosh\left(\frac{m\varphi}{2}\right)^{2/(\alpha+1)} + \cosh\left(\frac{m\varphi}{2}\right)^{-2/(\alpha+1)} \right] , \quad (6)$$

where  $V_0$  is a constant and  $m = 3(\alpha + 1)$ . In this case,  $c_s^2 = 1$ , irrespective of the value of  $\alpha$ .

In what follows, we shall discuss the observational bounds that can be set on the GCG model parameters.

### III. OBSERVATIONAL CONSTRAINTS

Given that the GCG model stands out as a potentially viable dark energy - dark matter unification scheme many authors have developed methods aiming to constrain its parameters from observational data, particularly through SNe Ia data [12], CMBR peak and trough location [2, 3] and amplitudes [13], and gravitational lensing statistics [5, 14]. More recent analysis based on the latest Type-Ia Supernova data has yielded rather surprising results, namely that  $\alpha > 1$ .

Particularly stringent constraints arise from the study of the position of the acoustic peaks and troughs of the CMBR power spectrum. The CMBR peaks arise from oscillations of the primeval plasma just before the Universe becomes transparent. Driving processes and the ensuing shifts on peak positions can be written as [15]

$$\ell_{p_m} \equiv \ell_A (m - \varphi_m) , \quad (7)$$

where  $\ell_A$  is the acoustic scale

$$\ell_A = \pi \frac{\tau_0 - \tau_{ls}}{\bar{c}_s \tau_{ls}} , \quad (8)$$

and with  $\tau_0$  and  $\tau_{ls}$  standing for the conformal time ( $\tau = \int a^{-1} dt$ ) today and at last scattering and  $\bar{c}_s$  the average sound speed before decoupling. Given that peak shift processes are fairly independent of physics after recombination they are not affected by the nature of the late time acceleration mechanism. Thus, the accurate fitting formulas of Ref. [16] can be used to compute the phase shifts  $\varphi_m$  for the GCG model. In order to estimate the acoustic scale, we use Eq. (2) and write the Universe rate of expansion as

$$H^2 = \frac{8\pi G}{3} \left[ \frac{\rho_{r0}}{a^4} + \frac{\rho_{b0}}{a^3} + \rho_{ch0} \left( A_s + \frac{(1 - A_s)}{a^{3(1+\alpha)}} \right)^{1/1+\alpha} \right] , \quad (9)$$

where  $A_s \equiv A/\rho_{ch0}^{1+\alpha}$ ,  $\rho_{ch0} \equiv (A + B)^{1/1+\alpha}$ . We have included the contribution of radiation and baryons as these are not accounted for by the GCG equation of state. As discussed in Refs. [2, 3], the above equations allow for obtaining the value of the fundamental acoustic scale by direct integration, using that  $H^2 = a^{-4} \left( \frac{da}{d\tau} \right)^2$ .

Comparison of the outcome from the above procedure with the most recent results on the location of the first two peaks and the first trough obtained by the WMAP collaboration [17], namely  $\ell_{p_1} = 220.1 \pm 0.8$ ,  $\ell_{p_2} = 546 \pm 10$ ,  $\ell_{d_1} = 411.7 \pm 3.5$ , with the bound on the position of the third peak obtained by the BOOMERanG collaboration [18],  $\ell_{p_3} = 825_{-13}^{+10}$ , gives origin to quite strong constraints on the model parameters. These constraints can be summarized as follows and critically depend on values of the spectral tilt,  $n_s$  and of the Hubble parameter,  $h$  [3, 4]:

1) Assuming WMAP priors, the Chaplygin gas model,  $\alpha = 1$ , is incompatible with the data and so are models with  $\alpha \gtrsim 0.6$ .

2) For  $\alpha = 0.6$ , consistency with data requires for the spectral tilt,  $n_s > 0.97$ , and that,  $h \lesssim 0.68$ .

3) The  $\Lambda$ CDM model barely fits the data for values of the spectral tilt  $n_s \simeq 1$  (WMAP data yields  $n_s = 0.99 \pm 0.04$ ) and for that  $h > 0.72$ . For low values of  $n_s$ ,  $\Lambda$ CDM is preferred to the GCG models, whereas for intermediate values of  $n_s$ , the GCG model is favoured only if  $\alpha \simeq 0.2$ .

These results are essentially consistent with the ones obtained in Refs. [13] using the CMBFast code. Furthermore, we find that:

4) Our study of the peak locations in the  $(A_s, \alpha)$  plane shows that, varying  $h$  within the bounds  $h = 0.71^{+0.04}_{-0.03}$  [17], does not lead to very relevant changes in the allowed regions, as compared to the value  $h=0.71$ , even though these regions become slightly larger as they shift up-wards for  $h < 0.71$ ; the opposite trend is found for  $h > 0.71$ .

5) Our results are consistent with bounds obtained in Ref. [2] using BOOMERanG data for the third peak and Archeops [19] data for the first peak as well as results from SNe Ia [12] and age bounds, namely  $0.81 \lesssim A_s \lesssim 0.85$  and  $0.2 \lesssim \alpha \lesssim 0.6$ .

6) If one abandons the constraint on  $h$  arising from WMAP, then the Chaplygin gas case  $\alpha = 1$  is consistent with the peaks location, if  $h \leq 0.64$ . [3].

Quite challenging, a new set of constraints arise from the latest SNe Ia data. These arise from the study 194 supernova data points from Ref. [20]. The results can be summarized as follows [6]:

7) Data favours  $\alpha > 1$ , although there is a strong degeneracy on  $\alpha$ . At 68% confidence level the minimal allowed values for  $\alpha$  and  $A_s$  are 0.78 and 0.778, thus ruling out the  $\Lambda$ CDM model  $\alpha = 0$  case. However, at 95% confidence level there is no constraint on  $\alpha$ .

8) If one does not assume the flat prior, one finds that GCG is consistent with data for values of  $\alpha$  sufficiently different from zero. Allowing some small curvature, positive or negative, one finds that the GCG model is a more suitable description than the  $\Lambda$ CDM model.

These results are analogous to the ones obtained in Refs. [21, 22] which find that the latest supernova data favours “phantom”-like matter with an equation of state of the form  $p = \omega\rho$  with  $\omega < -1$ .

## IV. CONCLUSIONS AND OUTLOOK

In this brief review we have outlined the way the GCG model allows for a consistent description of the accelerated expansion of the Universe and suggests an interesting and promising scheme for the unification of dark energy and dark matter. The model is quite detailed and its predictions can be directly confronted with observational data. For this purpose, several studies were performed aiming to constrain the parameter space of the model using Supernovae data, the age of distant quasar sources, gravitational lensing statistics and the location of the first few peaks and troughs the CMBR power spectrum, as measured by the WMAP and BOOMERanG collaborations. These studies reveal that a substantial portion of the parameter space of the GCG model can be excluded. In these studies it has been assumed that  $0 < \alpha \leq 1$ , however, a recent study of the latest supernova data indicates that  $\alpha > 1$  values are favoured. One can see that there is no contradiction between the various observational constraints at  $2\sigma$  level, even though a full analysis is still missing.

A critical question for the GCG model concerns structure formation. This is at the heart of the model as it is meaningful only as an entangled mixture of dark matter and dark energy. Concerns about this issue have been raised [23], however in this analysis the effect of baryons has not been taken into account, which was shown to be relevant and necessary for consistency with the 2DF mass power spectrum [24]. Furthermore, most computations were based on the linear treatment of perturbations close to the present time, thus neglecting any non-linear effects which are clearly important. Moreover, the role of entropy perturbations [25] in the non-linear regime and the comparison with observable quantities has to be further examined [26].

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